Math 2400 Practice Final Exam Version 1

Vanderbilt University

April 25, 2026

Name: ____

Please do not open the exam until instructed to do so.
You are allowed a non-CAS calculator and a notes sheet provided in class.
No phones, computers, smart watches, etc. are permitted.

The Vanderbilt Honor Code applies.

Question 1. (*10 points*) Let $y(t)$ be a solution to

$$t^2y'(t) + 2ty(t) = 2at, \quad \text{for } t > 0.$$

If we have

$$y(1) = 0, \quad y'(1) = 2,$$

find the constant a.

Question 2 *(10 points)* Solve the following nonlinear differential equation

$$\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$$

Question 3 *(10 points)* Initially, a tank contains 400L of water with 10 kg of salt in solution. Water containing 0.1 kg of salt per liter (L) is entering the tank at a rate of 1 L/min, and the mixture is allowed to flow out of the tank at a rate of 2 L/min. Let $Q(t)$ be the amount of salt at time t measured in kilograms. Write, but do not solve, a differential equation for $Q(t)$.

Question 4 *(10 points)* Let $A = PDP^{-1}$ be a 3×3 diagonalizable matrix where P is an invertible matrix and D is a diagonal matrix with diagonal entries $3, 0$ and -2. What are the eigenvalues of the matrix $A^3 - 2A$?

Question 5 *(10 points)* Solve the initial value problem

$$2xy + 2xy^2 + 1 + (x^2 + 2x^2y + 2y)\frac{dy}{dx} = 0, \quad y(1) = 2.$$

Problem 6 (*10 points*) Find the Laplace transform of $f(t) = \begin{cases} 0 & t < 2\pi \\ e^t \sin(t) & t > 2\pi \end{cases}$

Question 7. (*10 points*) If $\dfrac{du}{dx} = 3x^2u - u$ and $u(0) = 3$, find $u(2)$.

Question 8. (*15 points*) Let

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & 2 & 2 & 4 \\ 1 & 2 & 3 & 6 \\ 1 & 2 & 4 & 8 \end{pmatrix}$$

Find a basis for the null space, column space, and row space.

Question 9 *(10 points)* Let $A = \begin{pmatrix} -3 & -3 & -1 \\ 0 & 0 & 0 \\ 0 & -3 & -4 \end{pmatrix}$. Find all eigenvalues and a basis for each eigenspace.

Question 10. (*10 points*) On which interval is the initial value problem

$$\begin{cases} (5-t)y''(t) + (t-4)y'(t) + 2y(t) = \log(t) \\ y(2) = 8 \\ y'(2) = -4 \end{cases}$$

guaranteed to have a unique solution?

Question 11. (*10 points*) Find the general solution to the following inhomogeneous differential equation

$$y^{(4)}(t) - 3y^{(3)}(t) + 2y^{(2)}(t) = 4t - e^{t} + 3e^{3t}.$$

Problem 12. *(10 points)* Let $F(s) = \dfrac{6}{(s-3)^3}$ and $G(s) = \dfrac{5}{s^2+25}$. Calculate the inverse Laplace transform of $F(s)G(s)$.

Problem 13. (*15 points*) Solve the initial value problem

$$y''(t) + 4y'(t) + 5y(t) = \delta\left(t - \frac{\pi}{4}\right), \quad y(0) = 1,\ y'(0) = -2.$$

Problem 14. (*15 points*) Find the general solution to the following system

$$\vec{X}'(t) = \begin{pmatrix} 3 & 6 \\ -1 & -2 \end{pmatrix} \vec{X}(t).$$

Problem 15. (*10 points*) Let

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 0 & 3 \\ -3 & 2 & 1. \end{pmatrix}$$

Calculate A^{-1}.

Problem 16. (*15 points*) Find the inverse Laplace transform to the following transfer function

$$\frac{3s^2 + 4s - 1}{(s+1)(s^2 + 2s + 5)}$$

Problem 17. (*10 points*) Solve the following initial value problem

$$y'''(t) = f(t) \quad y''(0) = y'(0) = y(0) = 0.$$

End of Exam. Check your work!