

# Math 2400 Practice Exam 3

Vanderbilt University

April 8, 2026

Name: \_\_\_\_\_

Please do not open the exam until instructed to do so.  
You are allowed a non-CAS calculator and a notes sheet provided in class.  
No phones, computers, smart watches, etc. are permitted.

The Vanderbilt Honor Code applies.

**Question 1.** (*10 points*) Determine whether the following statements are true or false. If the statement is true, justify it. If the statement is false, provide a counterexample.

- A. If  $\lambda$  is an eigenvalue of a matrix  $A$ , then  $A - \lambda I$  is invertible.
- B. If an eigenvalue has geometric multiplicity less than its algebraic multiplicity, then the matrix is not diagonalizable.
- C. A matrix with a repeated eigenvalue is never diagonalizable.
- D. Consider  $x'' + 10x' + x = 0$ . The system is underdamped.

**Question 2** (*10 points*) A mass of 2 kg is attached to a spring with spring constant 8 N/m. A dashpot provides a damping force proportional to velocity with damping coefficient 6 N·s/m. The mass is pulled 0.5 meters to the right of equilibrium and released with an initial velocity of 1 m/s to the left. Set up the differential equation describing the motion of the mass and write the associated initial conditions.

**Question 3.** (15 points) Find the general solution to the differential equation

$$\frac{d^3y}{dx^3} + \frac{dy}{dx} = e^{-x} \cos(x) + x.$$

**Question 4** (10 points) Let  $L$  be a linear differential operator and suppose that  $\cos(t)$  and  $t$  are particular solutions to the differential equation

$$L[y] = f(x)$$

for a certain function  $f(x)$ . Find a solution to the homogeneous differential equation

$$L[y] = 0.$$

**Question 5** (10 points) Find the general solution to the differential equation

$$y'' + 3y' + 2y = e^{-t}$$

where  $y = y(t)$  subject to the initial conditions  $y(0) = 1$  and  $y'(1) = 1$ .

**Question 6.** (*10 points*) Find the general solution to the following system of differential equations

$$\begin{aligned}x'(t) &= x - 2y \\y'(t) &= 4x + 3y.\end{aligned}$$

**Question 7.** (*10 points*) Solve the second-order system

$$y''(t) + 7y'(t) + 10y(t) = 0$$

by writing it as a first order system of differential equations.

**Problem 8.** (10 points) Find the Jordan form for the following matrix:

$$\begin{bmatrix} 0 & 1 & 0 \\ -4 & 4 & 0 \\ -2 & 1 & 2 \end{bmatrix}$$

**Problem 9.** (15 points) Find the general solution to the differential equation

$$y''(t) - 2y'(t) + y(t) = \frac{e^t}{t^2 + 1}$$

*End of Exam. Check your work!*