Math 2400 Practice Exam 2

Vanderbilt University

March 4, 2026

Name: ______________________________

Please do not open the exam until instructed to do so.
You are allowed a non-CAS calculator and a notes sheet provided in class.
No phones, computers, smart watches, etc. are permitted.

The Vanderbilt Honor Code applies.

Question 1. (*20 points*) Determine whether the following statements are true or false. If the statement is true, justify it. If the statement is false, provide a counterexample.

A. If $AB = 0$, then either $A = 0$ or $B = 0$.

B. If two rows are proportional, then the determinant is zero.

C. Consider the vector space $V = C^2[a, b]$, all twice continuous functions on the interval $[a, b]$ and let $p(x)$ and $q(x)$ be smooth functions. The following space is a vector subspace of V:

$$H = \{y \in C^2[a, b] \mid y''(x) + p(x)y'(x) + q(x)y(x) = 0 \;\; \forall x \in [a, b]\}.$$

D. Consider the space of degree 3 polynomials $P_3(x)$. Define the *dot product* between vectors in $P_3(x)$ as

$$p(x) \cdot q(x) = \int_0^1 p(x)q(x)\, dx.$$

Define $W = \text{span}\{1, x\}$. Then, $W^\perp$ contains a polynomial of degree 1.

E. The dimension of the following subspace is 4

$$H = \text{span}\left\{ \begin{pmatrix} 1 \\ 2 \\ 4 \\ 8 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 4 \\ 12 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 0 \\ -4 \end{pmatrix} \right\}$$

F. Let A be a 5×9 matrix with $\text{rank}(A) = 4$. Which of the following statements must be true.

I. $\text{rank}(A^T) = 5$

II. $\dim(\text{row}(A)) = 4$

III. $\dim(\text{nul}(A)) = 1$

IV. $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions for every $\mathbf{b} \in \mathbb{R}^5$

V. The columns of A are linearly dependent

Question 2. (*10 points*) Let $A = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$. Find the matrix B that satisfies the following equation:

$$B^T A^{-1} = \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}$$

Question 3 *(10 points)* Let $A = \begin{pmatrix} 1 & 2 & 4 \\ 0 & 2 & 5 \\ 3 & 2 & 2 \end{pmatrix}$. For which values of r is the system $A\mathbf{x} = \begin{pmatrix} 2 \\ 3 \\ r \end{pmatrix}$ consistent?

Question 4 *(10 points)* Let $\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \right\}$ be the basis for the column space of a matrix A. Determine if the vector $\mathbf{x} = \begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix}$ is in the column space of A. If it is, write the vector $\mathbf{x}$ relative to the basis $\mathcal{B}$.

Question 5. (*10 points*) Which of the following set of vectors spans $\mathbb{R}^4$?

A. $\left\{ \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \right\}$

B. $\left\{ \begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -5 \\ -7 \\ 8 \\ 0 \end{pmatrix}, \begin{pmatrix} 5 \\ 6 \\ 6 \\ 3 \end{pmatrix} \right\}$

C. $\left\{ \begin{pmatrix} 3 \\ 2 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 7 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 5 \\ 1 \end{pmatrix} \right\}$

D. $\left\{ \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 6 \\ 5 \\ 0 \\ 3 \end{pmatrix} \right\}$

Question 6. (*10 points*) Let $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 3 & 4 & 5 \end{pmatrix}$. Calculate $\det(A)$.

Problem 7. (*10 points*) Write the solution as an augmented matrix and then solve it by computing the echelon form. To verify your solution, calculate the corresponding inverse matrix.

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\begin{aligned}
x_1 + 2x_2 + x_3 &= 4 \\
3x_1 + 8x_2 + 7x_3 &= 20 \\
2x_1 + 7x_2 + 9x_3 &= 23
\end{aligned}
$$

Problem 8 Let $A = \begin{pmatrix} 1 & 3 & -1 & 1 & 3 \\ 1 & 3 & 0 & 3 & 6 \\ -2 & -6 & 2 & -2 & -6 \end{pmatrix}$. Calculate the following:

1. The reduced row echelon form of A
2. The rank and nullity of A
3. A basis for the null space of A
4. A basis for the column space of A
5. A basis for the row space of A

End of Exam. Check your work!