Math 2400 Practice Exam 1

Vanderbilt University

February 4, 2026

Name: _____

Please do not open the exam until instructed to do so.
You are allowed a non-CAS calculator and a notes sheet provided in class.
No phones, computers, smart watches, etc. are permitted.

The Vanderbilt Honor Code applies.

Part 1. (*50 points*) Identify the following differential equations (separable, linear, Bernoulli, homogeneous, or exact) and then solve.

Q1 $\dfrac{dy}{dx} = 1 + x + y + xy$

Q2 $\dfrac{dy}{dx} = y + y^4$

Q3 $\dfrac{dy}{dx} = 3(y + 7)x^2$

Q4 $x\dfrac{dy}{dx} = y + \sqrt{x^2 - y^2}$

Q5 $(\cos(x) + \log(y))dx + \left(\dfrac{x}{y} + e^y\right) dy = 0$

Q6 Solve the following initial value problem.

$$\left(x^2 + 1\right) \frac{dy}{dx} + 3xy = x$$
$$y(0) = 1$$

Part 2. (*10 points*) Consider the autonomous first order ODE

$$\frac{dy}{dt} = y^2 - y$$

Q1 Find the equilibrium solutions.

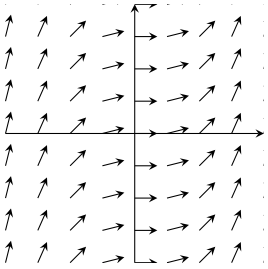
Q2 Determine the stability of the equilibrium solutions.

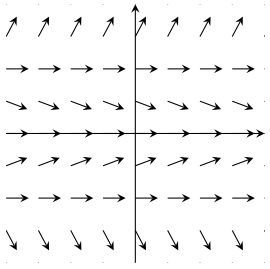
Q3 Sketch the solution curves.

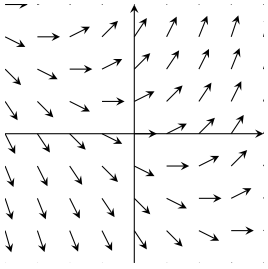
Part 3 (*10 points*)

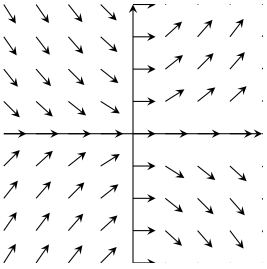
Q1 Match the following differential equations with their slope fields.

$$\frac{dy}{dx} = x + y \qquad \frac{dy}{dx} = y^3 - y \qquad \frac{dy}{dx} = x^2 \qquad \frac{dy}{dx} = y^{1/3}x^{1/3}$$

(A) 

(B) 

(C) 

(D) 

Q2 Which of the above ODEs fails the existence and uniquness theorem?

Part 4. (*10 points*) A tank initially contains 80 gal of brine containing 40 lb of salt. Brine containing 2 lb of salt per gallon enters the tank at the rate of 2 gal/min and the perfectly mixed brine in the tank flows out at the rate of 4 gal/min. Let $S(t)$ be the amount of salt at a time t in minutes. Write down the initial value problem for $S(t)$.

Part 5. (*10 points*) Use the elimination method to decide whether the following system is consistent or inconsistent. Then, find the solution set.

$$\begin{aligned} x - 2y + z &= 2 \\ 2x - y - 4z &= 13 \\ x - y - z &= 5 \end{aligned}$$

Part 6. (*10 points*) Write the solution as an augmented matrix and then solve it by computing the echelon form.

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 4 \\ 3x_1 + 8x_2 + 7x_3 &= 20 \\ 2x_1 + 7x_2 + 9x_3 &= 23 \end{aligned}$$

End of Exam. Check your work!