Final Exam Review

The final exam will take place on Saturday, April 24, 2025 from 3pm to 5pm in SC1320. You should go over the past three exams and determine which areas you need to review in the coming weeks. Here is a list of the topics and the aspects that you should prepare:

1. Exam 1 Content
   (a) Linear and nonlinear differential equations
      i. How to identify the difference between a linear and nonlinear differential equation.
      ii. The difference between an implicit and an explicit solution to a differential equation.

   (b) How to understand differential equations numerically and graphically
      i. Slope fields
      ii. Euler's method

   (c) Existence and Uniqueness

   (d) Tank problems

   (e) Solving first order differential equations
      i. Separable equations: an ODE of the form $\frac{dy}{dx} = f(x)g(x)$. To solve this, separate variables and integrate on both sides. Ex: $\frac{dy}{dx} = \frac{xe^{y+x^2}}{1+e^{2x^2}}$
      ii. Linear equations: an ODE of the form $\frac{dy}{dx} + p(x)y = q(x)$. To solve this, calculate the integrating factor $\mu = \exp \int p(x)dx$ and multiply by μ on both sides to yield a separable equation. Ex: $\frac{dy}{dx} = x + y$.
      iii. Homogeneous equation: An ODE of the form $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$. To solve this, make the substitution $v = \frac{y}{x}$ and calculate $\frac{dy}{dx} = \frac{dv}{dx}x + v$. Then solve the linear equation $\frac{dv}{dx}x + v = F(v)$. Ex: $x\frac{dy}{dx} = y + \sqrt{x^2 - y^2}$.
      iv. Bernoulli equation: an ODE of the form $\frac{dy}{dx} + p(x)y = q(x)y^n$. To solve this, divide by y^n on both sides and use the substitution $v = y^{1-n}$. Ex: $x\frac{dy}{dx} + 6y = 2xy^{4/3}$.
      v. Exact equation: an ODE of the form $M(x,y)dx + N(x,y)dy = 0$ where $\frac{dM}{dy} = \frac{dN}{dx}$. To solve this, check that the above criterion are satisfied. Then, find

the potential function $F(x, y)$ such that $\dfrac{\partial F}{\partial x} = M(x, y)$ and $\dfrac{\partial F}{\partial y} = N(x, y)$.
Ex: $(x + \arctan(y))\, dx + \left(\dfrac{x+y}{1+y^2}\right) dy = 0$

2. Exam 2 Content
   (a) Solving a system by Gaussian elimination
   (b) REF and RREF
   (c) Vector spaces
      i. Determining if something is a vector space using the axioms
      ii. Determining if something is a subspace by checking the three conditions.
   (d) Bases
      i. Determining a basis of a column space
      ii. Determining a basis of a row space
      iii. Determining a basis for the null space
   (e) Matrix algebra
      i. Adding and subtracting matrices
      ii. Multiplying two matrices
      iii. Finding the inverse of a matrix
      iv. Calculating combinations of the above manipulations
   (f) Determinants
      i. Calculating the determinant of a matrix using the reduction formula
      ii. Using the properties of determinants
   (g) Linear independence
      i. Using the definition of linear independence to show vectors are linear independent
      ii. Using the determinant to show linear independence
3. Exam 3 Content
   (a) Homogeneous higher order differential equations
      i. The superposition principle
      ii. Calculating characteristic polynomials for constant coefficient differential equations
         A. Real distinct roots
         B. Repeated roots
         C. Complex roots
      iii. Finding solutions to the initial value problem using linear algebra
   (b) Inhomogeneous higher order differential equations

i. The complex exponential. Understand the equation $e^{iax} = \cos(ax) + i\sin(ax)$ and its consequences.
ii. Solving the equation $y''(t) + p\,y'(t) + q\,y(t) = f(t)$
   A. $f(t) = e^{at}$ (exponential response): If a is not a root to the characteristic polynomial $p(r)$, the particular solution is $y_p = \dfrac{e^{at}}{p(a)}$.
   B. $f(t) = e^{at}$ (resonance response): If a is a root to the characteristic polynomial $p(r)$, the particular solution is $y_p = \dfrac{te^{at}}{p'(a)}$
   C. $f(t) = \cos(at)$ or $f(t) = \sin(at)$: Write $\cos(at) = \Re(e^{iat})$ and $\sin(at) = \Im(e^{iat})$ and use the exponential response formula.
   D. $f(t) =$ polynomial: use undetermined coefficients.
   E. An arbitrary $f(t)$: use the variation of parameters.
iii. Calculating eigenvalues and eigenvectors
iv. Determining the geometric multiplicity of an eigenvalue
v. Diagonalization of a matrix
   A. Finding the diagonalization
   B. Calculating powers of a matrix
vi. Jordan form
   A. Calculating the Jordan form
   B. Finding the entries in the matrix P.
vii. Solving systems of differential equations
   A. Two real eigenvalues
   B. Complex eigenvalues
viii. Writing second-order ODEs as a first-order system.

4. Content after Exam 3

(a) **Solving a 2×2 system of differential equations with a repeated eigenvalue of geometric multiplicity one**

(b) The Laplace transform
   i. Calculating the Laplace transform of basic functions ($e^{at}, \sin(at), \cos(at)$, etc)
   ii. Calculating the inverse Laplace transform using partial fraction decompositions
   iii. Solving differential equations with a smooth input using the Laplace transform

(c) Piecewise functions
   i. Writing piecewise functions in terms of the unit step function
   ii. Calculating the Laplace transforms of functions that involve the unit step function

   iii. Calculating the inverse Laplace transform of transfer functions with exponentials.

(d) Solving differential equations with piecewise input

(e) Solving differential equations with a unit impulse input

(f) Solving differential equations using the Convolution