Solutions HW6

① a) $x - 2y + 5z = 0$

$$(x, y, z) = (2y - 5z, y, z)$$

$$= y(2, 1, 0) + z(-5, 0, 1)$$

$$B = \{(2,1,0)^T, (-5,0,1)^T\}.$$

b) $y = z$

$$(x, y, z) = (x, y, y)$$

$$= x(1,0,0) + y(0,1,1)$$

$$B = \{(1,0,0)^T, (0,1,1)^T\}.$$

c) line intersection between (a) & (b)

$$\begin{aligned} x - 2y + 5z &= 0 \\ y &= z \end{aligned} \quad \Rightarrow \quad x + 3y = 0$$

$$x = -3z = -3y$$

$$(x, y, z) = (-3z, z, z)$$

$$= z(-3, 1, 1)$$

$$B = \{(-3,1,1)^T\}.$$

② a) Is $x^2 - x^3 \in \text{span}\{x^2, 2x+x^2, x+x^3\}$

$$x^2 - x^3 = ax^2 + b(2x+x^2) + c(x+x^3)$$
$$= ax^2 + 2bx + bx^2 + cx + cx^3$$
$$= (2b+c)x + (a+b)x^2 + cx^3$$

$c = -1$

$a + b = 1 \Rightarrow a = 1/2$

$2b + c = 0 \Rightarrow 2b = 1, \ b = 1/2$

b) $\{x^2, 2x+x^2, x+x^3\}$

Writing these as column vectors.

$(0,1,0)$ $(2,1,0)$ $(1,0,1)$

These three vectors are linearly independent.

c) dimension is three

d) Need constant terms

Add constant 1.

(3) $A = \begin{pmatrix} 1 & 2 & -1 & 0 \\ 2 & 4 & -2 & 1 \\ -1 & -2 & 1 & 3 \end{pmatrix}$

$$\xrightarrow{\text{RREF}} \begin{pmatrix} 1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

a) Basis for the row space.

$B = \{(1,2,-1,0), (0,0,0,1)\}$

b) Basis for column space.

$B = \{(1,2,-1)^T, (0,1,3)^T\}$

c) Basis for null space.

pivot columns are 1 and 4

$x_1 + 2x_2 - x_3 = 0$

$x_4 = 0$

set $x_2 = t$, $x_3 = s$

$(x_1, x_2, x_3, x_4) = (-2t+s, t, s, 0)$

$= t(-2,1,0,0) + s(1,0,1,0)$

$B = \{(-2,1,0,0), (1,0,1,0)\}.$