First Order ODE Practice Questions

1. Solve the following differential equations

   (a) $x\dfrac{dy}{dx} = y + 2\sqrt{xy}$

(b) $x^2 \dfrac{dy}{dx} = 1 - x^2 + y^2 - x^2 y^2$

(c) $\dfrac{d^2y}{dx^2} = \sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}$ (Hint: This equation does not involve y.)

(d) $\dfrac{dy}{dx} = e^{4x-7y}$

(e) $x\dfrac{dy}{dx} = y + x$

(f) $\dfrac{dy}{dx} + \cot(x)y = \cos(x)$

(g) $x\dfrac{dy}{dx} = y + \sqrt{x^2 - y^2}$

(h) $x\dfrac{dy}{dx} + 6y = 3xy^{4/3}$

(i) $\dfrac{dy}{dx} = -\dfrac{(1+y^2)(x+\arctan(y))}{x+y}$

(j) $(e^x \sin(y) + \tan(y))dx + (e^x \cos(y) + x \sec^2(y))dy = 0$

(k) $3xy^2\dfrac{dy}{dx} = 3x^4 + y^3$

(l) $\dfrac{dy}{dx} + \dfrac{x}{x^2+1}y = 1$

(m) $\dfrac{dy}{dx} + \dfrac{x}{x+1}y = \dfrac{e^{-x}(x^3+x^2)}{x^2+1}$

2. Consider the differential equation

$$\frac{dy}{dx} = x + y \qquad y(0) = 0.$$

(a) Construct a slope field for the differential equation.

(b) In your above slope field, sketch the solution curve corresponding to the initial data.

(c) Use the above curve to estimate the value of $y(-4)$.

3. Consider the differential equation

$$\frac{dy}{dx} = 2026\sqrt{y}.$$

Explain why the two solutions $y_1(x) = 1026169x^2$ and $y_2(x) = 0$ are two solutions that satisfy the initial condition $y(0) = 0$.

4. Consider the autonomous differential equation

$$\frac{dy}{dt} = y^3 - y.$$

(a) Find the equilibrium solutions to the differential equation.

(b) Determine the stability of the equilibrium solutions in part (a).

(c) Sketch the solution curves for the differential equation.

5. A 400 gallon tank initially contains 100 gallons of brine containing 50 lb of salt. Brine containing 1 lb of sale per gallon enters the tank at a rate of 5 gal/s, and the well-mixed brine in the tank flows out at a rate of 3 gal/s. How much salt will be in the tank when it is full of brine?

6. A tank contains 1000 liters of a solution consisting of 100 kg of salt dissolved in water. Pure water is pumped into the tank at a rate of 5 L/s, and the mixture– kept uniform by stirring– is pumped out at the same rate. How long will it be until only 10 kg of salt remain in the tank.

7. For what values of α are the following differential equations exact?

   (a) $(y\cos(3xy) + \alpha x)dx + (\alpha x\cos(2xy) - 2y)dy = 0$

   (b) $-y(x^2 + y^2)^\alpha dx + x(x^2 + y^2)^\alpha dy = 0$

8. Consider a population $P(t)$ satisfying the following initial value autonomous differential equation

$$\frac{dP}{dt} = 0.01P(100 - P), \qquad P(0) = P_0.$$

Which of the following statement(s) is/are true?

(a) If $P_0 > 0$, then $\lim_{t\to\infty} P(t) = 100$.

(b) If $P_0 \neq 100$, then $P(t) \neq 100$ for all $t > 0$.

(c) If $0 < P_0 < 100$, then we can find a $t > 0$ such that $P(t) = 0$.