

Exam 2 Practice Problems

1. Use the integral comparison test to show if the following integrals converge or diverge.

(a) $\int_1^{\infty} \frac{1}{x^{2026} + 2026} dx$

(b) $\int_1^{\infty} \frac{1}{x + e^{2x}} dx$

2. Determine whether the following series converge. State what tests you are using and verify the hypotheses.

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{4^{n+1}}$

(b) $\sum_{n=1}^{\infty} \frac{n+2}{\sqrt{n^4+1}}$

(c) $\sum_{n=3}^{\infty} \frac{\ln(n)}{n^3}$

(d) $\sum_{n=1}^{\infty} \frac{n^n}{(n!)^2}$

(e) $\sum_{n=3}^{\infty} \frac{1}{(\ln(n))^{\ln(n)}}$

(f) $\sum_{n=1}^{\infty} \ln\left(\frac{n^5}{5n^5 + 4n^3 + 1}\right)$

(g) $\sum_{n=1}^{\infty} \left(\frac{n}{n^4 - 3n^2 + 3^n}\right)^{2n}$

(h) $\sum_{n=2}^{\infty} \frac{\ln(\ln(n))}{n \ln(n)}$

(i) $\sum_{n=2}^{\infty} \frac{\cos(n)}{n^5 + n^3 + 2}$

(j) $\sum_{n=1}^{\infty} (-1)^{n-1} (\ln(n+1) - \ln(n))$

(k) $\sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{n^n}$

(l) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{2n-1}$

3. Determine the radii and the intervals of convergence for the following power series.

(a) $\sum_{n=1}^{\infty} \frac{n^5}{5^n n!} (x-2)^n$

(b) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln(n)} (x-1)^n$

(c) $\sum_{n=1}^{\infty} \frac{x^n}{3^n n}$

(d) $\sum_{n=1}^{\infty} 2^n n^2 x^n$

(e) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln(n)} x^n$

(f) $\sum_{n=1}^{\infty} n! (2x-1)^n$

(g) $\sum_{n=1}^{\infty} \frac{b^n}{\ln(n)} (x-a)^n \quad b > 0$

4. Determine whether each of the statements below is true or false. If it is true, justify your response. If it is false, provide a counterexample.

(a) If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both diverge, then $\sum_{n=1}^{\infty} (a_n + b_n)$ also diverges.

(b) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} (a_n)^2$ also converges.

(c) If $a_n > 0$ and the series $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} n a_n$ diverges.

(d) If $a_n > 0$ and the series $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} \frac{a_n}{n}$ converges.

(e) If $a_n > 0$ and the series $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} \sin(a_n)$ converges.

(f) If $\sum_{n=1}^{\infty} a_n < \sum_{n=1}^{\infty} b_n$ and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ also diverges.

(g) If $\sum_{n=1}^{\infty} a_n$ conditionally converges, then $\sum_{n=1}^{\infty} |a_n|$ must diverge.

- (h) If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both diverge, then $\sum_{n=1}^{\infty} a_n b_n$ also diverges.
- (i) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\lim_{n \rightarrow \infty} a_n = 0$.
- (j) If $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} (a_n + b_n)$ converges.
- (k) The 37th partial sum for the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + 5}$ is an overestimate.
- (l) Every convergent series must pass the ratio test.
- (m) If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge, then $\sum_{n=1}^{\infty} a_n b_n$ also converges.
- (n) If the series $\sum_{n=1}^{\infty} a_n$ converges and $a_n > 0$, then $\sum_{n=1}^{\infty} (a_n)^2$ converges.

5. Find a power series representation for the following functions.

- (a) $f(x) = \frac{x-1}{x+2}$
- (b) $g(x) = \frac{x^2+1}{(1-x)^2}$
- (c) $h(x) = \frac{\ln(3-5x)}{x^2}$
- (d) $q(x) = x \cos\left(\frac{x}{2}\right)$

6. Calculate the value of the following series

- (a) $\sum_{n=1}^{\infty} \frac{6^{n+2}}{3^{2n}}$
- (b) $\sum_{n=0}^{\infty} \frac{5^{n+1} + (-3)^n}{6^n}$
- (c) $\sum_{n=0}^{\infty} \frac{n+1}{2^n}$
- (d) $\sum_{n=0}^{\infty} \frac{3^n}{5^n n!}$