Power Series Practice Problems

Determine the interval of convergence of the following power series.

1. $\sum_{n=1}^{\infty} \frac{x^n}{3^n n}$

2. $\sum_{n=1}^{\infty} 2^n n^2 x^n$

3. $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln(n)} x^n$

4. $\sum_{n=1}^{\infty} n!(2x-1)^n$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{b^n}{\ln(n)}(x-a)^n \qquad b > 0$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{n!x^n}{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}$

Find the power series representation of the following functions and determine their intervals of convergence.

1. $f(x) = \dfrac{x-1}{x+2}$

2. $g(x) = \dfrac{x^2 + x}{(1 - x)^3}$

3. $h(x) = \dfrac{\ln(3-5x)}{x^2}$

4. $q(x) = x \cos\left(\frac{x^2}{2}\right)$

5. $w(x) = \sin^2(x)$

6. $p(x) = \tanh^{-1}(x)$ Hint: $\tanh^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$