Math 1301 Practice Midterm 3 Version 2

Vanderbilt University

1 October 2025

Name: ______________________________

Please do not open the exam until instructed to do so.
You are allowed a one-page (double-sided) formula sheet.
Your instructor may ask to see your formula sheet.
No calculators, phones, computers, smart watches, etc. are permitted.

The Vanderbilt Honor Code applies.

Part 1. (*25 points*) Determine if the following series converge or diverge. Show your work and clearly state any convergence divergence tests you use.

Q1 $\displaystyle\sum_{n=0}^{\infty} \frac{5000^n}{n! \sin^8(n)}$

Q2 $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n^4 - 3n^2 + 3^n}\right)^{2n}$

Q3 $\displaystyle\sum_{n=2}^{\infty} \frac{\ln(\ln(n))}{n \ln(n)}$

Q4 $\displaystyle\sum_{n=2}^{\infty} \frac{\cos(n)}{\sqrt{3n-5}}$

Q5 $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{n^n}$

Q6 $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{2n-1}$

Part 2. (*10 points*) Find the interval of convergence for the following power series.

$$p(x) = \sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln(n)} (x-1)^n$$

Part 3 (*10 points*) Determine if the following are true or false. If the statement is false, provide a counter-example; if the statement is true, try to justify your answer.

Q1 If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} -a_n$ converge, then $\sum_{n=1}^{\infty} |a_n|$ converges.

Q2 If $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty}(a_n + b_n)$ converges.

Q3 If $\lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right| < 1$, then $\sum_{n=1}^{\infty} a_n$ converges.

Q4 The sum of the telescoping series $\sum_{n=1}^{\infty}(a_n - a_{n+1})$ is a_1.

Q5 The 37th partial sum for the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+5}$ is an overestimate.

Q6 Every convergent series must pass the ratio test.

Q7 If $\sum_{n=1}^{\infty} a_n$ converges absolutely, then $\sum_{n=1}^{\infty} \frac{a_n}{1+a_n}$ converges.

Q8 If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge, then $\sum_{n=1}^{\infty} a_n b_n$ also converges.

Q9 A power series always converges at at least one point.

Q10 If the series $\sum_{n=1}^{\infty} a_n$ converges and $a_n > 0$, then $\sum_{n=1}^{\infty} (a_n)^2$ converges.

Part 4. (*15 points*) Showing work is not required but partial credit may be awarded if you do.

Q1 The value of $\displaystyle\sum_{n=0}^{\infty} \frac{45^{n+1}}{3^{4n} n!}$ is

Q2 The number of terms to estimate $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$ with an error of less than $\dfrac{1}{100}$ is

Q3 Use power series to evaluate following limit.

$$\lim_{x \to 0} \frac{\sin(x) - x}{x^3}$$

Part 5. (*20 points*) Showing work is not required but partial credit may be awarded if you do.

Q1 Find a series representation for π using $\tan^{-1}(x)$.

Q2 Find the first four terms in the Taylor series centered at $x = 1$ for $g(x) = 5^x$.

Part 6. (*10 points*) Use power series to evaluate the integral

$$\int \ln(x)\arctan(x)dx.$$

Hint: Use integration by parts first.

Part 7. (*10 points*) Prove the identity $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ in the following steps:

Q1 Find the power series representation of $p(x) = \frac{\sin(\sqrt{x})}{\sqrt{x}}$.

Q2 Find the zeros of $p(x)$.

Q3 Use the following fact to prove the result: *For a polynomial* $p(x) = a_n x^n + .. + a_1 x + a_0$ whose roots are $r_1, ..., r_n$,

$$\frac{1}{r_1} + ... + \frac{1}{r_n} = -\frac{a_1}{a_0}.$$

End of Exam. Check your work!