Math 1301 Practice Midterm 3 Version 1

Vanderbilt University

1 October 2025

Name: ______________________________

Please do not open the exam until instructed to do so.
You are allowed a one-page (double-sided) formula sheet.
Your instructor may ask to see your formula sheet.
No calculators, phones, computers, smart watches, etc. are permitted.

The Vanderbilt Honor Code applies.

Part 1. (*25 points*) Determine if the following series converge or diverge. Show your work and clearly state any convergence divergence tests you use.

Q1 $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{4^{n+1}}$

Q2 $\displaystyle\sum_{n=1}^{\infty} \frac{n+2}{\sqrt{n^4+1}}$

Q3 $\displaystyle\sum_{n=3}^{\infty} \frac{\ln(n)}{n^3}$

Q4 $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{(n!)^2}$

Q5 $\displaystyle\sum_{n=3}^{\infty} \frac{1}{(\ln(n))^{\ln(n)}}$

Q6 $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n^5}{5n^5 + 4n^3 + 1}\right)$

Part 2. (*10 points*) Find the interval of convergence for the following power series.2

$$p(x) = \sum_{n=1}^{\infty} \frac{n^5}{5^n n!}(x-2)^n$$

Part 3 (*20 points*) Determine if the following are true or false. If the statement is false, provide a counter-example; if the statement is true, try to justify your answer.

Q1 If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

Q2 If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} (-1)^n a_n$ also diverges.

Q3 If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{\pi}{e}$, then $\sum_{n=1} a_n$ diverges.

Q4 If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both diverge, then $\sum_{n=1}^{\infty} (a_n + b_n)$ also diverges.

Q5 If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} (a_n)^2$ also converges.

Q6 If $\sum_{n=1}^{\infty} a_n < \sum_{n=1}^{\infty} b_n$ and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ also diverges.

Q7 If $\sum_{n=1}^{\infty} a_n$ conditially converges, then $\sum_{n=1}^{\infty} |a_n|$ must diverge.

Q8 If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both diverge, then $\sum_{n=1}^{\infty} a_n b_n$ also diverges.

Q9 If $\sum_{n=1}^{\infty} a_n$ diverges, then $\lim_{n\to\infty} a_n = 0$.

Q10 The series $\sum_{n=1}^{\infty} \frac{(-1)^n \cos(n\pi)}{n^2}$ converges by the alternating series test.

Part 4. (*15 points*) Showing work is not required but partial credit may be awarded if you do.

Q1 The value of $\sum_{n=1}^{\infty} \frac{6^{n+2}}{3^{2n}}$ is [ ]

Q2 The value of $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$ is [ ]

Q3 The domain of the function $f(p)$ is [ ] where $f(p)$ is given by

$$f(p) = \sum_{n=1}^{\infty} \frac{\ln(1+n^p)}{n^p}$$

Part 5. (*20 points*) Showing work is not required but partial credit may be awarded if you do.

Q1 Find a power series representation for $f(x) = \dfrac{x-5}{(1-x)^2}$ and determine its radius of convergence.

Q2 Find the first four terms in the Taylor series centered at $x = 3$ for $g(x) = \sin^2(x)$.

Part 6. (*10 points*) Showing work is not required but partial credit may be awarded if you do.

Q1 Find the power series representation of $h(x) = \ln(1 + x^2)$.

Q2 Using your solution to Q1, determine the value of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n(2n+1)}$. (*Hint:* $\int_0^1 x^{2n} = \frac{1}{2n+1}$)

End of Exam. Check your work!